

# Mark Scheme (Results) January 2008

GCE

GCE Mathematics (6674/01)

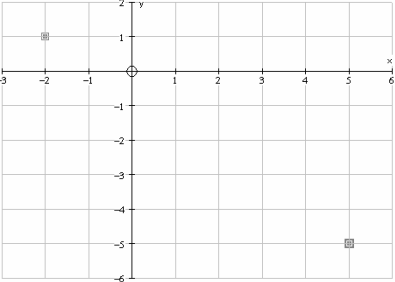
January 2008  
6674 Further Pure Mathematics FP1  
Mark Scheme

Question Number	Scheme	Marks
1	<p>Integrating factor = <math>e^{-3x}</math></p> $\therefore \frac{d}{dx}(ye^{-3x}) = xe^{-3x}$ $\therefore (ye^{-3x}) = \int xe^{-3x} dx = -\frac{x}{3}e^{-3x} + \int \frac{1}{3}e^{-3x} dx$ $= -\frac{x}{3}e^{-3x} - \frac{1}{9}e^{-3x} (+c)$ $\therefore y = -\frac{x}{3} - \frac{1}{9} + ce^{3x}$	B1 M1 M1 A1 A1ft <b>[5]</b>
	<p>Notes:</p> <p>First M for multiplying through by Integrating Factor and evidence of calculus</p> <p>Second M for integrating by parts ‘the right way around’. Be generous – ignore wrong signs and wrong constants.</p> <p>Second M dependent on first. Both As dependent on this M.</p> <p>First A1 for correct expression – constant not required</p> <p>Second A requires constant for follow through.</p> <p>If treated as a second order de with errors then send to review.</p>	

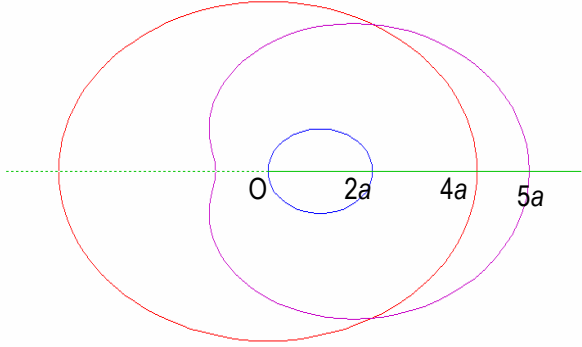
<p>2.</p>	<p>Use <math>(2x+1)</math> as factor to give <math>f(x) = (2x+1)(x^2 - 6x + 10)</math></p> <p>Attempt to solve quadratic to give <math>x = \frac{6 \pm \sqrt{(36-40)}}{2}</math></p> <p>Two complex roots are <math>= 3 \pm i</math></p>	<p>M1 A1</p> <p>M1 A1</p> <p>M1 A1</p> <p>(6)</p> <p>[ 6 ]</p>
	<p>Notes:</p> <p>First M if method results in quadratic expression with 3 terms (even with remainder).</p> <p>Second M for use of correct formula on their quadratic.</p> <p>Third M for using <math>i</math> from negative discriminant.</p>	
<p>3.</p> <p>(a)</p> <p>(b)</p>	<p>Consider <math>\frac{(x+3)(x+9) - (3x-5)(x-1)}{(x-1)}</math>, obtaining <math>\frac{-2x^2 + 20x + 22}{(x-1)}</math></p> <p>Factorise to obtain <math>\frac{-2(x-11)(x+1)}{(x-1)}</math>.</p> <p>Identify <math>x = 1</math> and their two other critical values</p> <p>Obtain one inequality <i>as an answer</i> involving at least one of their critical values</p> <p>To obtain <math>x &lt; -1, 1 &lt; x &lt; 11</math></p>	<p>M1 A1</p> <p>M1 A1</p> <p>(4)</p> <p>B1ft</p> <p>M1</p> <p>A1, A1</p> <p>(4)</p> <p>[ 8 ]</p>
	<p>Notes:</p> <p>Second M attempt to factorise quadratic expression with 3 terms (usual rules).</p> <p>Second A don't require -2 outside but can be part of factors.</p>	

<p>4. (a)</p> <p>(b)</p>	<p><math>f(0.7) = -0.195028497</math> and <math>f(0.8) = 0.297206781</math></p> <p>Use <math>\frac{0.8 - \alpha}{\alpha - 0.7} = \frac{f(0.8)}{-f(0.7)}</math> to obtain <math>\alpha = \frac{-0.8f(0.7) + 0.7f(0.8)}{f(0.8) - f(0.7)}</math></p> <p><math>(=0.739620991) = 0.740</math> Answer required to 3 dp or better</p> <p><math>f'(x) = 6x + 1 - \frac{1}{2}\sec^2\left(\frac{x}{2}\right)</math></p> <p>Use <math>x_2 = 0.75 - \frac{f(0.75)}{f'(0.75)}</math> (<math>= 0.741087218</math>) <math>= 0.741</math> Answer required to 3 dp or better</p>	<p>B1, B1</p> <p>M1</p> <p>A1</p> <p>(4)</p> <p>M1 A1</p> <p>M1 A1</p> <p>(4)</p> <p>[ 8 ]</p>
	<p>Notes:</p> <p>(a) Bs for 3dp or better</p> <p>First M for reasonable attempt using fractions and differences.</p> <p>(b) First M attempt to differentiate <math>f(x)</math>, term in <math>x</math> is enough.</p> <p>Lose last A if either or both not to 3 dp</p>	

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<p>5. (a)</p> <p>(b)</p>	<p>Method to obtain partial fractions e.g. <math>5r + 4 = A(r + 1)(r + 2) + Br(r + 2) + Cr(r + 1)</math> And equating coefficients, or substituting values for <math>x</math>.</p> <p><math>A = 2, B = 1, C = -3</math> or <math>\frac{2}{r} + \frac{1}{r+1} - \frac{3}{r+2}</math></p> $\sum_{r=1}^n \dots = \frac{2}{1} + \frac{1}{2} - \frac{3}{3}$ $+ \frac{2}{2} + \frac{1}{3} - \frac{3}{4}$ $+ \frac{2}{3} + \frac{1}{4} - \frac{3}{5} \quad = \quad 2 + \frac{3}{2}, -\frac{2}{n+1} - \frac{3}{n+2} \text{ or equivalent}$ $+ \dots$ $+ \frac{2}{n-1} + \frac{1}{n} - \frac{3}{n+1}$ $+ \frac{2}{n} + \frac{1}{n+1} - \frac{3}{n+2}$ $= \frac{7(n+1)(n+2) - 4(n+2) - 6(n+1)}{2(n+1)(n+2)} = \frac{7n^2 + 11n}{2(n+1)(n+2)} *$	<p>M1</p> <p>A1 A1 A1 (4)</p> <p>M1 A1, A1</p> <p>M1 A1 (5)</p>
	<p>Notes:</p> <p>(a) Require three constants for method.</p> <p>(b) Require first 3 and last 2 of their terms for first method</p> <p>Second method - dependent on first - for attempt to combine to single fraction.</p> <p>Expansion of <math>(n+1)(n+2)</math> in numerator and correct solution required for final A1</p>	

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<p>6(a)</p> <p>(b)</p> <p>(c)</p> <p>(d)</p>	<p>( i ) Multiply top and bottom by conjugate to give <math>\frac{-2-i}{5}</math></p> <p>( ii ) Expand and simplify using <math>i^2 = -1</math> to give <math>3 - 4i</math></p> <p><math>z^2 - z = 5 - 5i,  z^2 - z  = 5\sqrt{2} *</math></p> <p><math>\arg(z^2 - z) = -\frac{\pi}{4}</math> or <math>-45^\circ</math> or <math>7\pi/4</math> or <math>315^\circ</math> or <math>-0.7853\dots</math> or <math>5.497\dots</math></p>  <p>one mark for each point</p>	<p>M1 A1</p> <p>M1 A1</p> <p>(4)</p> <p>M1A1</p> <p>(2)</p> <p>M1 A1</p> <p>(2)</p> <p>B1,</p> <p>B1 ft</p> <p>(2)</p> <p><b>[10]</b></p>
	<p>Notes:</p> <p>(a) <math>-2-i</math> or <math>2+i</math> OK for method. Attempt to expand required.</p> <p>(b) square root required for method</p> <p>(c) 2 for correct answer only, tan required for method. 2dp or better.</p> <p>(d) Position of points not clear but both quadrants correct first B1 only.</p>	

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7 (a)	<p>Solve auxiliary equation <math>3m^2 - m - 2 = 0</math> to obtain <math>m = -\frac{2}{3}</math> or 1</p> <p>C.F is <math>Ae^{-\frac{2}{3}x} + Be^x</math></p> <p>Let PI = <math>\lambda x^2 + \mu x + \nu</math>. Find <math>y' = 2\lambda x + \mu</math>, and <math>y'' = 2\lambda</math> and substitute into d.e.  Giving <math>\lambda = -\frac{1}{2}</math>, <math>\mu = \frac{1}{2}</math> and <math>\nu = -\frac{7}{4}</math></p> <p><math>\therefore y = -\frac{1}{2}x^2 + \frac{1}{2}x - \frac{7}{4} + Ae^{-\frac{2}{3}x} + Be^x</math></p> <p>(b) Use boundary conditions:  <math>2 = -\frac{7}{4} + A + B</math>  <math>y' = -x + \frac{1}{2} - \frac{2}{3}Ae^{-\frac{2}{3}x} + Be^x</math> and <math>3 = \frac{1}{2} - \frac{2}{3}A + B</math></p> <p>Solve to give <math>A = 3/4</math>, <math>B = 3</math> (<math>\therefore y = -\frac{1}{2}x^2 + \frac{1}{2}x - \frac{7}{4} + \frac{3}{4}e^{-\frac{2}{3}x} + 3e^x</math>)</p>	<p>M1 A1</p> <p>A1ft</p> <p>M1</p> <p>A1 A1A1</p> <p>A1ft</p> <p><b>(8)</b></p> <p>M1A1ft</p> <p>M1 M1</p> <p>M1 A1</p> <p><b>(6)</b></p> <p><b>[14]</b></p>
	<p>Notes:</p> <p>(a) Attempt to solve quadratic expression with 3 terms (usual rules)</p> <p>Both values required for first accuracy.</p> <p>Real values only for follow through</p> <p>Second M 3 term quadratic for PI required</p> <p>Final A1ft for their CF+ their PI dependent upon at least one M</p> <p>(b) Second M for attempt to differentiate their y and third M for substitution</p>	

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<p><b>8</b> (a)</p> <p>(b)</p> <p>(c)</p>	<p><math>a(3 + 2 \cos \theta) = 4a</math> Solve to obtain <math>\cos \theta = \frac{1}{2}</math> <math>\theta = \pm \frac{\pi}{3}</math> and points are <math>(4a, \frac{\pi}{3})</math> and <math>(4a, \frac{5\pi}{3})</math></p> <p>Use area = <math>\frac{1}{2} \int r^2 d\theta</math> to give <math>\frac{1}{2} a^2 \int (3 + 2 \cos \theta)^2 d\theta</math> Obtain <math>\int (9 + 12 \cos \theta + 2 \cos 2\theta + 2) d\theta</math> Integrate to give <math>11\theta + 12 \sin \theta + \sin 2\theta</math> Use limits <math>\frac{\pi}{3}</math> and <math>\pi</math>, then double or <math>\frac{\pi}{3}</math> and <math>\frac{5\pi}{3}</math> or theirs Find a third area of circle = <math>\frac{16\pi a^2}{3}</math> Obtain required area = <math>\frac{38\pi a^2}{3} - \frac{13\sqrt{3}a^2}{2}</math></p>  <p>correct shape 5a and 4a marked 2a marked and passes through O</p>	<p>M1 M1 A1, A1 (4)</p> <p>M1 A1 M1 A1 M1 B1 A1, A1 (8)</p> <p>B1 B1 B1 (3)</p> <p><b>[15]</b></p>
	<p>Notes:</p> <p>(a) First A for <math>r=4a</math> second for both values in radians. Accept 1.0471... and 5.2359... 2 dp or better for final A</p> <p>(b) First M for substitution, expansion and attempt to use double angles. Second M for integrating expression of the form <math>a + b \cos \theta + c \cos 2\theta</math> Lose final A only if <math>a^2</math> missing in last line</p> <p>(c) First B for approximately symmetrical shape about initial line, only 1 loop which is convex strictly within shaded region</p>	